7. What are # of possible soccer formations (D – M – F) if there are at least 1 ~~of~~ each type out of 10 players?

slots - persons

10 slots __ __ __ __ __ __ __ __ __ __

If any choice is possible there would be

$$\underset{D,M,F}{\underline{3}} \cdot \underline{3} \cdot \underline{3} \cdots \underline{3} = 3^{10} \text{ possibilities.}$$

We don't want to count situations where there are only 1 position (3) all D or all M or all F

or 2 positions.

$$\left.\begin{array}{l} \underset{D \text{ or } M}{\underline{2}} \cdots 2 = 2^{10} \\ \underset{M \text{ or } F}{2} \cdots = 2^{10} \\ \underset{D \text{ or } F}{2} \cdots = 2^{10} \end{array}\right\} \text{Only 2 positions or only 1 position}$$

overcounting by factor of 2

$$\text{Answer: } \underset{\text{all poss}}{3^{10}} - \Big(\underset{\text{(2 or 1 positions)}}{(2^{10})3} - \underset{\text{only 1 position}}{3}\Big) = \boxed{55{,}980}$$

overcounting

This is the

Next method

1+	1+	1+
D	M	F

# of ways to place 10 individuals into the various positions D/M/F, where at least 1 is in each

# of different formations is

$$\left| \begin{array}{c|c|c} 1+ & 1+ & 1 \\ A+ & B+ & C \end{array} \right| \quad \text{where } A+B+C=7$$

D M F

We get

again this will not be exactly what we want.

Method to solve this: (Bars & Stars)

Players D|M|F

* * * | * * | * * * * *

3D, 2M, 5F

Count # of these things

* * * * | * * | * * * *

4D | 2M | 4F

=

* | * | *

1D + 1M 1F

extra

# of possibilities

$$\underbrace{* * \,|\, * * * \,|\, * *}_{+2D \quad +3M \quad +2F}$$

$$\begin{array}{c|c|c} D & M & F \\ 3 & 4 & 3 \end{array}$$

$$* \quad * \quad * \,|\,|\, * \quad * \quad * \quad *$$

+3D   +0M   +4F

D	M	F
4	1	5

→ # of possibilities

9 slots $\underline{*}\ \underline{*}\ \underline{|}\ \underline{*}\ \underline{*}\ \underline{*}\ \underline{|}\ \underline{*}\ \underline{*}$

→ # of ways to put 2 of them as bars (rest stars)

$$\#\text{ of ways} = \binom{9}{2} = \frac{9\cdot 8}{2\cdot 1} = \boxed{36} = \frac{9!}{2!\,7!}$$

General formula for Stars & Bars.

# of ways to place k bars among n stars

$$= \binom{n+k}{k} = \frac{(n+k)!}{k!\,n!}$$

$$\left.\begin{matrix} 1 & 1 & 8 \\ 1 & 2 & 7 \\ & \vdots & \\ 8 & 1 & 1 \end{matrix}\right\} 36 \text{ ways.}$$

Cool staff about combinations.

$$(A+B)^n = \underbrace{(A+B)(A+B)(A+B)\dots(A+B)}_{n \text{ times}}$$

$$(A+B)(A+B) = A\cdot A + B\cdot A + A\cdot B + B\cdot B$$

$$(A+B)(A+B)(A+B) \qquad = A^2 + 2AB + B^2$$

$$= A\cdot A\cdot A + A\cdot A\cdot B + A\cdot B\cdot A + B\cdot A\cdot A$$

$$+ B\cdot B\cdot A + B\cdot A\cdot B + A\cdot BB + B\cdot B\cdot B.$$

$$= A^3 + \underline{3}A^2B + \underline{3}AB^2 + B^3$$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

Pascal's triangle

$$(A+B)^5 = A^5 + 5A^4B + 10A^3B^2 + 10A^2B^3 + 5AB^4 + B^5$$

$$1 = \binom{5}{0} \qquad \binom{5}{1} \qquad \binom{5}{2} = \frac{5\cdot 4}{2\cdot 1} = 10 \qquad \binom{5}{3} \qquad \binom{5}{4} \qquad 1 = \binom{5}{5}$$

Binomial formula Thm

$$\boxed{(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^{n-k} B^k}$$